

Precise determination of the η_c mass and width in the radiative $J/\psi \rightarrow \eta_c \gamma$ decay

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Abstract. We present an effective field theory based extraction of the η_c mass and width from a recent measurement by CLEO of the photon line shape in the $J/\psi \rightarrow \eta_c \gamma$ decay.

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We study the radiative decay $J/\psi \rightarrow \eta_c \gamma$ within an effective field theory (EFT) framework, namely potential non-relativistic QCD (pNRQCD) [1, 2, 3]. Our motivation is to take advantage of the new measurements made by the CLEO [4] and KEDR [5] collaborations of the photon line shape, in order to obtain precise determinations of the η_c mass and width (for a recent review see [6]). Magnetic dipole transitions in quarkonium were studied in pNRQCD in Ref. [7], where, for the $J/\psi \rightarrow \eta_c \gamma$ decay, a branching ratio consistent, within errors, with the PDG value [8] was found. We apply the same formalism here. Before discussing it, however, we briefly review the CLEO and KEDR analyses.

To fit its data [4], CLEO uses two background sources,

- a Monte Carlo modeled background for spurious $J/\psi \rightarrow X$ with shape

$$\text{bkg}_1(E_\gamma) = N \left(e^{-5.720 E_\gamma} + 10.441 e^{-33.567 E_\gamma} \right)$$

- and a freely fit background for $J/\psi \rightarrow \pi^0 X$ and non-signal $J/\psi \rightarrow X$ with shape

$$\text{bkg}_2(E_\gamma) = A + B E_\gamma + C E_\gamma^2,$$

and a theoretical line shape given by

$$\text{theory}(E_\gamma) = E_\gamma^3 \times \text{BW}_{\text{rel}}(E_\gamma) \times \text{damping}(E_\gamma),$$

where the relativistic Breit–Wigner distribution is

$$\begin{aligned} \text{BW}_{\text{rel}}(E_\gamma)^{-1} &= \left(M_{J/\psi}^2 - 2M_{J/\psi} E_\gamma - M_{\eta_c}^2 \right)^2 \\ &\quad + \left(M_{J/\psi}^2 - 2M_{J/\psi} E_\gamma \right) \Gamma_{\eta_c}^2, \end{aligned}$$

$M_{J/\psi}$ and M_{η_c} stand for the masses of the J/ψ and η_c respectively, Γ_{η_c} for the η_c width, and

$$\text{damping}(E_\gamma) = e^{-E_\gamma^2/(8\beta^2)}.$$

The damping function accounts for the overlap of the two quarkonium states, assumed to be described by wavefunctions of an harmonic oscillator, and the photon. The natural scale of β is then the typical momentum transfer inside the charmonium ground state, which is about 700 to 800 MeV. This implies almost no damping for photon energies smaller than 500 MeV, which is consistent with the multipole expansion of the electromagnetic fields. The value that comes from CLEO's fit is, however, an order of magnitude smaller, $\beta = (65.0 \pm 2.5)$ MeV, which implies the vanishing of the signal for photon energies of few hundred MeV. The CLEO analysis yields the values $M_{\eta_c} = (2982.2 \pm 0.6)$ MeV and $\Gamma_{\eta_c} = (31.5 \pm 1.5)$ MeV.

The η_c line shape is also studied by the KEDR collaboration in [5]. Their analysis is similar to CLEO's one, but an alternative damping function is used as well:

$$\text{damping}'(E_\gamma) = \frac{E_{\text{peak}}^2}{E_\gamma E_{\text{peak}} + (E_\gamma - E_{\text{peak}})^2},$$

where E_{peak} is the most probable transition energy. If CLEO's data are used, the analysis of KEDR gives $M_{\eta_c} = (2982.4 \pm 0.7)$ MeV and $\Gamma_{\eta_c} = (32.5 \pm 1.8)$ MeV when fitting with $\text{damping}(E_\gamma)$, and $M_{\eta_c} = (2981.8 \pm 0.5)$ MeV and $\Gamma_{\eta_c} = (33.6 \pm 1.9)$ MeV when fitting with $\text{damping}'(E_\gamma)$. However, an analysis of KEDR's own preliminary data gives different values: $M_{\eta_c} = (2979.7 \pm 1.6)$ MeV and $\Gamma_{\eta_c} = (26.9 \pm 4.8)$ MeV when fitting with $\text{damping}(E_\gamma)$, and $M_{\eta_c} = (2979.4 \pm 1.5)$ MeV and $\Gamma_{\eta_c} = (27.8 \pm 5.1)$ MeV when fitting with $\text{damping}'(E_\gamma)$.¹ The discrepancy between different val-

¹ Incidentally, the KEDR results are closer to previous η_c mass measurements from J/ψ and $\psi(2S) \rightarrow \eta_c \gamma$ decays (averaging (2977.3 ± 1.3) MeV [9]), while the CLEO value is in agreement with the re-

ues of the η_c mass and width is larger than the experimental sensitivity, which highlights the importance of performing a critical analysis of the theory inputs.

Potential NRQCD exploits the hierarchy of scales in the problem and allows to express physical observables as systematic expansions in the ratio of these scales. Heavy quark-antiquark bound states are characterized by a number of scales: the heavy-quark mass, the typical momentum, $\langle p \rangle$, exchanged by the quarks (this is also of the order of the inverse of the typical size of the bound state, $1/\langle r \rangle$), the binding energy, the typical hadronic scale Λ_{QCD} , and possibly other smaller scales. In the transition $J/\psi \rightarrow \eta_c \gamma$, the relevant scales are the charm mass m_c , which is much larger than the next-to-largest scale, which is $\langle p \rangle \sim 1/\langle r \rangle \sim 800$ MeV, which in turn is larger than Λ_{QCD} . The binding energy of the J/ψ is $E_{J/\psi} \sim 500$ MeV, which is smaller than $1/\langle r \rangle$. There are also two smaller scales that have to be considered in addition: the hyperfine splitting, which is $M_{J/\psi} - M_{\eta_c} \sim 120$ MeV and smaller than $E_{J/\psi}$, and the width of the η_c , $\Gamma_{\eta_c} \sim 30$ MeV, which is smaller than the hyperfine splitting. In the radiative decay $J/\psi \rightarrow X \gamma$ around the η_c peak, we consider photon energies that vary between 0 MeV and $500 \text{ MeV} < 1/\langle r \rangle$. Under these conditions, we may describe the charmonium ground state in weakly coupled pNRQCD (because the system is non-relativistic and the typical momentum transfer is larger than Λ_{QCD}), couple the electromagnetic fields to pNRQCD and multipole expand the electromagnetic fields (because the photon energy is smaller than the typical momentum transfer in the bound state).

Three main processes contribute to $J/\psi \rightarrow X \gamma$ within pNRQCD in the energy range of interest ($0 \text{ MeV} \leq E_\gamma \leq 500 \text{ MeV}$):

- the M1 transition $J/\psi \rightarrow \eta_c \gamma \rightarrow X \gamma$,
- the E1 transitions $J/\psi \rightarrow \chi_{c(0,2)}(1P) \gamma \rightarrow X \gamma$,
- fragmentation and other background processes included in the background functions.

The evaluation of the magnetic dipole contribution yields (see Fig. 1)

$$\frac{d\Gamma_{J/\psi \rightarrow \eta_c \gamma}^{\text{M1}}}{dE_\gamma} = \frac{64}{27} \frac{\alpha}{\pi} \frac{E_\gamma^3}{M_{J/\psi}^2} \frac{\Gamma_{\eta_c}/2}{(M_{J/\psi} - M_{\eta_c} - E_\gamma)^2 + \frac{\Gamma_{\eta_c}^2}{4}}. \quad (1)$$

It has been pointed out in Ref. [4] that the dependence on E_γ^3 is responsible for the asymmetric shape of the photon spectrum.

sults obtained in $\gamma\gamma$ fusion and $p\bar{p}$ production (averaging (2982.6 ± 1.0) MeV [9]).

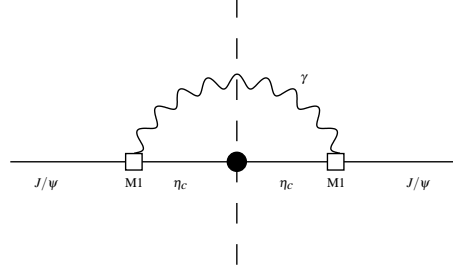


FIGURE 1. M1 contribution to the η_c line shape. The black dot stands for four-fermion operators that contribute to the decay width of the intermediate state.

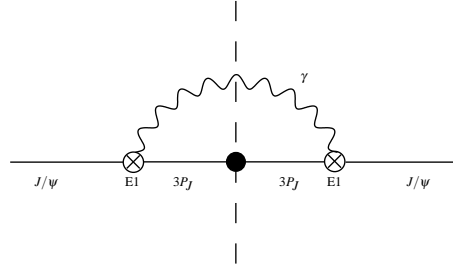


FIGURE 2. E1 contribution to the η_c line shape.

The electric dipole contribution is (see Fig. 2)

$$\frac{d\Gamma_{J/\psi \rightarrow \eta_c \gamma}^{\text{E1}}}{dE_\gamma} = \frac{448}{243} \alpha \frac{E_\gamma}{m_c} \alpha_s^2 \frac{|\phi_{J/\psi}(0)|^2}{m_c^3} |a_e(E_\gamma)|^2, \quad (2)$$

where $\phi_{J/\psi}(0)$ is the J/ψ wavefunction at the origin. The function $a_e(E_\gamma)$ has been discussed in Refs. [10, 11]; a closed analytical form has been derived in [12].

In the weak-coupling regime, the typical momentum transfer in the charmonium is of order $m_c \alpha_s$, the binding energy of the ground state is of order $m_c \alpha_s^2$ and the hyperfine splitting is of order $m_c \alpha_s^4$. The magnetic and electric dipole contributions are of equal order for $m_c \alpha_s \gg E_\gamma \gg m_c \alpha_s^2$. The magnetic contribution completely dominates the electric one in the peak region ($m_c \alpha_s^2 \gg E_\gamma \gg m_c \alpha_s^4$) and it also dominates by a factor $E_{J/\psi}^2 / (M_{J/\psi} - M_{\eta_c})^2 \sim 1/\alpha_s^4$ for $E_\gamma \ll m_c \alpha_s^4$.

We use as theoretical line shape the sum of Eqs. (1) and (2), and as background the sum of $\text{bkg}_1(E_\gamma)$ and $\text{bkg}_2(E_\gamma)$. As predicted by the power counting and numerically confirmed, for photon energies ≤ 500 MeV, the electric dipole contribution is negligible with respect to the magnetic one. The signal shape has been convolved with a Gaussian resolution function, whose resolution width is 4.8 MeV [4]. Our best fit is shown in Fig. 3. The fitting parameters are M_{η_c} , Γ_{η_c} , an overall normalization, the signal normalization, and the background parameters

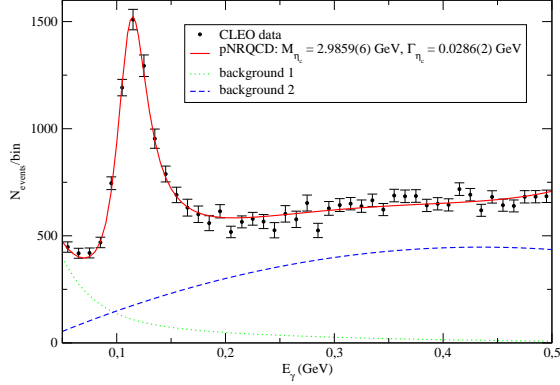


FIGURE 3. Our best fit to CLEO’s data for the photon spectrum in $J/\psi \rightarrow \eta_c \gamma$ using Eqs. (1) and (2) for the theoretical signal together with the two background sources $\text{bkg}_1(E_\gamma)$ and $\text{bkg}_2(E_\gamma)$.

A, B and C. In particular, the best fit line shape parameters for the η_c are (errors are only statistical)

$$\begin{aligned} M_{\eta_c} &= (2985.9 \pm 0.6) \text{ MeV}, \\ \Gamma_{\eta_c} &= (28.6 \pm 0.2) \text{ MeV}. \end{aligned}$$

The main differences between our analysis and the one of CLEO are summarized in the following.

- At leading order in the multipole expansion of the photon field, which is justified in the range $E_\gamma \leq 500$ MeV, and in the non-relativistic limit, the overlap integral in the magnetic dipole transition is equal to one. This amounts to setting equal to one the damping function in CLEO’s analysis. We have checked that higher-order terms in the multipole expansion are indeed negligible. Our conclusion is that CLEO’s damping function has no theoretical justification as an overlap integral, which is also signalled by the unnatural scale of the parameter β . The absence of a damping function accounts for about 50% of the difference in the η_c mass determination in our analysis and in that one of CLEO.
- Another difference is that in our analysis the differential width turns out to be proportional to a non-relativistic Breit–Wigner distribution ($\propto E_\gamma^3$), see Eq. (1), while CLEO uses a relativistic Breit–Wigner distribution. The relativistic Breit–Wigner distribution resums some classes of relativistic corrections, without including others of the same size. Partial resummations could lead to spurious effects, while the use of a non-relativistic Breit–Wigner distribution appears to be more justified in the framework of a systematic relativistic expansion, like the one provided by pNRQCD. This difference accounts for about the remaining 50% difference in the determination of the η_c mass.

In summary, radiative decays of quarkonia and specifically the transition $J/\psi \rightarrow \eta_c \gamma$ may be investigated in an EFT framework that systematically exploits the hierarchy of scales in the system (pNRQCD). The total transition width has been calculated in [7] including the next-to-leading order relativistic corrections. Within a large theoretical uncertainty that determination is in agreement with the experimental value. In this work, we consider the photon line shape in the non-relativistic limit. We obtain a best fit, which is in good agreement with CLEO’s experimental determination, see Fig. 3. However, theoretical errors have not been included so far in our analysis and we expect them to be larger than those coming from the fitting accuracy. For instance, relativistic corrections could impact the η_c mass by corrections as large as those induced by the difference between the relativistic and the non-relativistic Breit–Wigner distribution. Under investigation is also the extraction of the $J/\psi \rightarrow \eta_c \gamma$ branching ratio from the photon spectrum.

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REFERENCES

1. N. Brambilla, A. Pineda, J. Soto and A. Vairo, *Rev. Mod. Phys.* **77** (2005) 1423.
2. A. Pineda and J. Soto, *Nucl. Phys. Proc. Suppl.* **64** (1998) 428.
3. N. Brambilla, A. Pineda, J. Soto and A. Vairo, *Nucl. Phys. B* **566** (2000) 275.
4. R. E. Mitchell *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **102** (2009) 011801.
5. V. V. Anashin *et al.*, [arXiv:1002.2071](https://arxiv.org/abs/1002.2071) [hep-ex].
6. N. Brambilla *et al.*, [arXiv:1010.5827](https://arxiv.org/abs/1010.5827) [hep-ph].
7. N. Brambilla, Y. Jia and A. Vairo, *Phys. Rev. D* **73** (2006) 054005.
8. K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).
9. C. Amsler *et al.* [Particle Data Group], ”Review of particle physics,” *Phys. Lett. B* **667** (2008) 1.
10. A. V. Manohar and P. Ruiz-Femenía, *Phys. Rev. D* **69** (2004) 053003; P. D. Ruiz-Femenía, *Nucl. Phys. Proc. Suppl.* **152** (2006) 200.
11. P. D. Ruiz-Femenía, *Nucl. Phys. B* **788** (2008) 21, *PoS EFT09* (2009) 005.
12. M. B. Voloshin, *Mod. Phys. Lett. A* **19** (2004) 181.